Fermilab

PBAR NOTE #455

Transverse Beam Heating Distributions

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Transverse Beam Heating Distributions

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A beam is heated in one dimension with a dipole kicker which deflects the beam by an angle $\theta(t)$, at a location where the lattice functions are (α,β,γ). θ has a power spectrum $P(\omega)$ in the sense that θ^2 represents power. We will use variables I,γ defined as

$$I = (\gamma x^2 + 2\alpha x x' + \beta x'^2)/2$$

$$\tan\gamma = \alpha + \beta x'/x \qquad 1.$$

When a particle is deflected by an angle θ, I and γ change by

$$\delta I = (2I\beta)^{1/2}\sin\gamma\,\theta + \beta\theta^2/2$$

$$\delta\gamma = (2\beta/I)^{1/2}\cos\gamma\,\theta \qquad 2.$$

Although γ will diffuse, we are only interested in amplitudes I. To use the Fokker-Planck (FP) equation, we need to calculate the sum over many revolutions of the δI's ($=\Delta I$) for a particle, and find those parts of ΔI and $(\Delta I)^2$ which are proportional to time. The particle, which has (angular) revolution frequency ω_0 and betatron frequency $\omega_\beta = \omega_0\nu$ passes the kicker at times t_ℓ with betatron phase γ_ℓ where

$$t_\ell = t_o + 2\pi\ell/\omega_o$$

$$\gamma_\ell = \gamma_o + \omega_\beta t_\ell \qquad 3.$$

The final values of ΔI and $(\Delta I)^2$ are then averaged over t_o and γ_o. We find;

$$\Delta I/T = \beta\omega_o\int P(\omega)d\omega/2\pi \equiv k_w$$

$$(\Delta I)^2/T = \beta I\omega_o^2\Sigma_{p=0}^{\infty}P(|p\omega_o \pm \omega_\beta|)/2\pi \equiv 2kI \qquad 4.$$

The FP equation is

$$\partial f/\partial t + \partial\Phi/\partial I = 0$$

$$\Phi = k_w f - kI\partial f/\partial I = 0 \qquad 5.$$

For narrow band heating, the first term can be neglected, since its contribution to FP is of order BW/f_o compared to that of the second term. For wideband heating $BW>f_o$, it changes the nature of the diffusion, and the subsequent distribution. Eq. 5 admits solutions, where C is a constant,

$$f = Ce^{-\alpha t}J_o(2\sqrt{(\alpha I/k)}) \qquad 6.$$

Now f must be 0 at, as well as beyond, the aperture limit I_m; otherwise its infinite derivative there would imply infinite flux. Then $J_o(2\sqrt{(\alpha I_m/k)}) = 0$, and the allowed values of α are

$$\alpha_i = kj_i^2/4I_m \qquad 7.$$

Here j_i is the i<u>th</u> zero of J_0. Since the set of functions $J_0(j_i x)$ is complete in the interval $(0,1)$, the initial distribution can be expressed as a series of $J_0(j_i\sqrt{(I/I_m)})$. Each term then decays with its own lapse rate α_i. For our purposes, we note that all terms above $i=1$ decay very rapidly compared to $i=1$, so the distribution quickly approaches $J_0(j_1\sqrt{(I/I_m)})$.

What one measures with the scraper is the integral distribution $F(y)$ as a function of y, where $y = \sqrt{(I\beta_s)}$, and β_s is the value at the scraper. $F(y)$ is given by

$$F(y) = yJ_1(j_1 y/y_m)/[y_m J_1(j_1)] \qquad 8.$$

This can be compared directly to the plots on the Lexidata.